Widrow Hoff Learning Algorithm Based Minimization of BER Multi-User Detection in SDMA

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***Abstract*—In this paper minimization of BER in MUD based on neural network has been proposed. The change in weights from Widrow-Hoff learning algorithm has been used to update the weight vectors of the equalizer. Neural networks can be used for linear design, adaptive prediction, amplitude detection, character recognition and many other applications. In this paper adaptive prediction has been used in detecting the errors caused in AWGN channel. These errors are rectified by using Adaptive prediction methods based LMS algorithm for updating their weights. SDMA scheme with 3 users and 4 receiver antennas has been considered in the present work for obtaining the results. BPSK is used as the modulation scheme.**

***Index Terms*— Adaptive Algorithm, bit error rate (BER), channel, neural networks, multi user detection (MUD), Widrow-Hoff.**

1. INTRODUCTION

Neural Network based smart antennas are capable of improving the achievable wireless system capacity and quality by suppressing the effects of both inter-symbol interference (ISI) and co-channel interference (CCI). In this paper, we consider a space-division multiple access (SDMA) uplink scheme, where each transmitter employs a single antenna, while the base station (BS) receiver has multiple antennas [1-3]. In a CDMA system, each user is separated by a unique user-specific spreading code. By contrast, an SDMA system differentiates each user by the associated unique user specific channel impulse response (CIR) encountered at the receiver antennas. In this analogy, the unique user-specific CIR plays the role of a user-specific CDMA signature. However, owing to the non-orthogonal nature of the CIRs, an effective multiuser detection (MUD) is required for separating the users in an SDMA system [4-7]. Neural networks have recently been used in the design of multiuser receivers for SDMA systems**.** Neural Network based receivers employing the Widrow-Hoff criterion usually show good performance and have simple adaptive implementation, at the expense of a higher computational complexity [8-11]. The deployment of non-linear structures, such as neural networks, can mitigate more effectively inter-symbol interference, caused by the multipath effect of radio signals, and multiple access interference, which arise due to the non-orthogonality between user signals. In the last few years, different artificial neural networks structures have been used in the design of multiuser detectors (MUD). These neural systems make use of non-linear functions to create decision boundaries to detect transmitted symbols, whilst conventional (MUDs) employ linear functions to form such decision regions. In addition, the bulk of previously reported neural and linear receivers are based upon the LMS criterion, since this approach usually shows good performance and has simple adaptive implementation**.** However, it is well known that the LMS cost function is not optimal in digital communications applications, and the most appropriate cost function is the minimum bit error rate (MBER)**.** The approximate minimum bit error rate (AMBER) is one of the most successful and suitable algorithms for adaptive implementation using linear receiver structures, provided the application can handle a long training sequence. However, the AMBER methodology has not been proposed for SDMA networks and hence the present work throws light on this area. In this paper, a similar approximate minimum bit error rate approach to adaptive multiuser receivers using dynamic neural networks based on Widrow-Hoff learning algorithm has been proposed.

The paper is organized as follows. Section II briefly describes the SDMA system model and a linear multiuser receiver. The proposed methodology of Widrow-Hoff algorithm is explained in section III. Section IV is dedicated to minimizing the BER using Widrow-Hoff learning algorithm. Section V shows the discusses and the simulated results and conclusion are drawn in section VI.

2. SYSTEM MODEL

Consider a MIMO system employing M users with a single transmit antenna each and as entire receiver assisted by an L-element antenna array, shown in the Fig 1. For the M users supported in the system, the transmitted samples at the symbol rate are $s_m(k)=[s_m(k)\ s_m(k+1)\ s_m(k+2)\ \ldots]^T$, $1\leq m\leq M$, where $s_m(k)$ is the k^{th} transmitted symbol of the user m. At the L receive antennas, Gaussian white noises are introduced. For the

l^{th} receive antenna, $n_l(k)$, an independently identically distributed complex-valued Gaussian white noise process with $E[n_l(k)] = 0$ and $E[|n_l(k)|^2] = 2\sigma_n^2$ is added to the noise-free part of the l^{th} receive antenna's output $\overline{x_l}(k)$. Then the received signal samples $x_l(k)$ at the symbol rate for $1 \le l \le L$ are given

detectors has the same decision delay d, and all the $0 \le d \le n_F + n_c - 2$. Let us define

$$w_m = [w_{1,m}^T w_{2,m}^T \cdots w_{L,m}^T]^T \quad (4)$$

$$x_l(k) = [x_l(k) x_l(k-1) \cdots x_l(k - n_F + 1)]^T \quad (5)$$

$$x(k) = [x_1^T(k) x_2^T(k) \cdots x_L(k)]^T \quad (6)$$

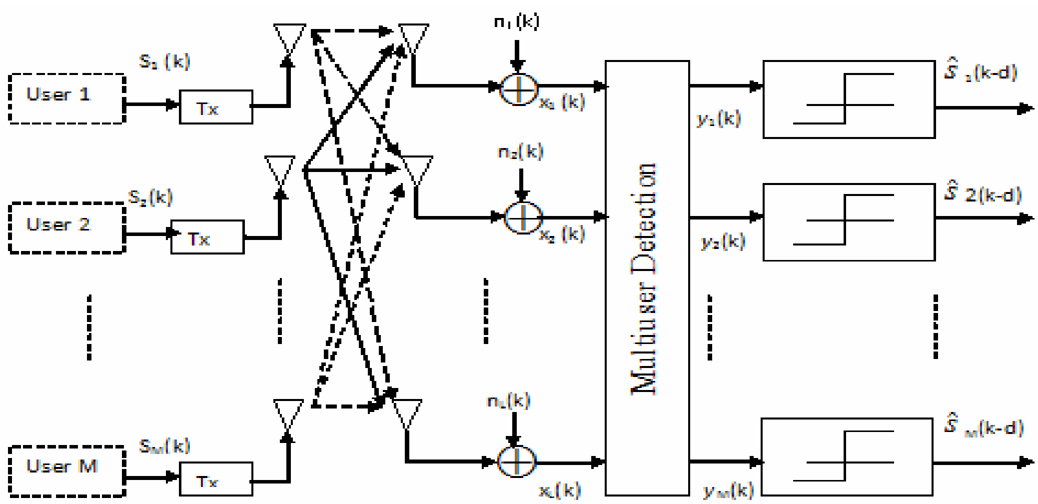

Fig. 1 Schematic of an antenna-array-aided SDMA system, where each of the M users is equipped with a single transmit antenna, and the receiver is assisted by an L-element antenna array.

$$x_l(k) = \sum_{m=1}^{M} \sum_{i=0}^{n_C - 1} c_{i,l,m} s_m(k-i) + n_l(k) = \overline{x_l}(k) + n_l(k) \quad (1)$$

where $c_{l,m} = [c_{0,l,m}\ c_{1,l,m} \cdots c_{n_C-1,l,m}]^T$ denotes the tap vector of the CIR connecting the user m and the l^{th} receive antenna . In BPSK modulation each user $s_m(k) \in \{\pm 1\}$ has the equal transmit power of $E[|s_m(k)\}^2] = \sigma_s^2 = 1$.

$$y_m(k) = \sum_{l=0}^{L} \sum_{i=0}^{n_F - 1} w_{i,l,m}^* x_l(k-i) \quad (2)$$

for $1 \le m \le M$,

where $w_{l,m} = [w_{0,l,m} w_{1,l,m} \cdots w_{n_{F-1},l,m}]^T$

denotes the m^{th} user detector's equalizer weight vector associated with the l^{th} receive antenna. The M user detectors' decisions are defined by

$$\hat{s}_m(k-d) = \text{sgn}(y_{R_m}(k)), 1 \le m \le M \quad (3)$$

where $\hat{s}_m(k-d)$ is the estimate of $s_m(k-d)$, $y_{Rm}(k) = \Re[y_m(k)]$ denotes the real part of $y_m(k)$, and $\text{sgn}(\bullet)$ the sign function. Again for notational simplicity, we assume that each of the M

Then the output of the m^{th} detector can be written as

$$y_m(k) = \sum_{l=1}^{L} w_{l,m}^H x_l(k) = w_m^H x(k) \quad (7)$$

Let us define the $n_F \times (n_F + n_C - 1)$ CIR convolution matrix associated with the user m and l^{th} receive antenna as shown in the bottom of the page, and further introduce the overall system CIR convolution matrix as

For $1 \le m \le M$, where $w_{l,m} = [w_{0,l,m} w_{1,l,m} \cdots w_{n_{F-1},l,m}]^T$ denotes

$$C = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,m} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{l,1} & c_{l,2} & \cdots & c_{l,m} \end{bmatrix} \quad (8)$$

The received signal vector $x(k)$ can be expressed by

$$x(k) = Cs(k) + n(k) = \overline{x}(k) + n(k) \quad (9)$$

where

$$n(k) = [n_1(k) n_2(k) \cdots n_L(k)]^T \quad (10)$$

With $n_l(k) = [n_l(k) n_l(k-1) \cdots n_L(k - n_F + 1)]^T$ and

$$s(k) = [s_1^T(k) s_2^T(k) \cdots s_M^T(k)]^T \quad (11)$$

With

$$s_m(k) = [s_m(k) s_m(k-1) \cdots s_M(k - n_F - n_C + 2)]^T$$

note that the output of the m^{th} detector can be expressed as

$$s(k) = [s_1^T(k) s_2^T(k) \cdots s_M^T(k)]^T \qquad (12)$$

With

$$s_m(k) = [s_m(k) s_m(k-1) \cdots s_M(k - n_F - n_C + 2)]^T$$

note that the output of the m^{th} detector can be expressed as

$$y_m(k) = w_m^H(\bar{x}(k) + n(k)) = \bar{y}_m(k) + e_m(k) \qquad (13)$$

where $e_m(k)$ is Gaussian distribution function having zero mean.

since it minimizes the squared error between the desired output value d_i and the neutron's activation value $net_i = w_i^t(x)$. The learning signal for this rule is defined as follows

$$r \triangleq d_i - w_i^t x \qquad (14)$$

The weight vector increment under this learning rule is

$$\Delta w_i = c(d_i - w_i^t x)w$$

or, for the single weight the adjustment is

$$\Delta w_{i,j} = c(d_i - w_i^t x)w_j \quad \text{for } j=1,2, \ldots\ldots, n$$

This rule can be considered as a special case of the delta learning rule. Indeed, assuming that the activation function is simply the identity function becomes identity to (1). This rule is sometimes called LMS (least mean square) learning rule. Weights are initialized at any values in this method. Thus the Widrow-Hoff methodology can be used for obtaining the change in weight values. Here, the weight vectors corresponding to equalizers have been considered.

$$c_{l,m} = \begin{bmatrix} c_{0.l.m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} & 0 & \cdots & 0 \\ 0 & c_{0.l.m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} & \cdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & c_{0.l.m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} \end{bmatrix}$$

3. WIDROW-HOFF ALGORITHM

Artificial neural systems computation lies in the middle ground between engineering and artificial intelligence. The neural network can also be defined as interconnection of neurons, such neuron outputs are connected through weights, to all other neurons including them. Both lag-free and delay connections are allowed. A network can be connected in cascade to create a multilayer network. In such network, the output of a layer is the input to the following layer is called FEED FORWARD. A feedback network can be obtained from the feed-forward network by connecting the neurons outputs to their inputs.

In the early 1960s a device called ADALINE (for ADAptive LINEar combiner) was introduced and a new powerful learning rule called the Widrow-Hoff learning rule was developed by BERNARD Widrow and Marcian Hoff (1960, 1962). This method falls under feedback network category. The rule minimized the summed square error during training involving pattern classification.

The Widrow-Hoff learning rule is applicable for the supervised training of neural networks. It is independent of the activation function of neurons used

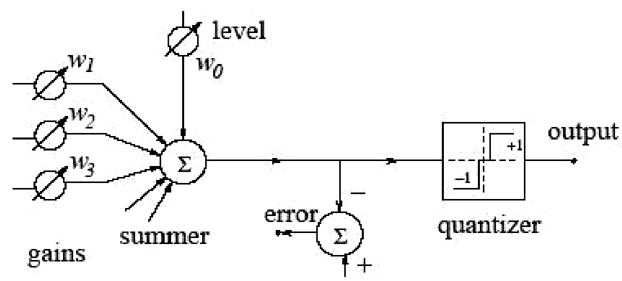

Fig. 2 Widrow –Hoff Learning algorithm

4. MINIMIZING THE BER USING WIDROW-HOFF LEARNING ALGORITHM

Step: 1 Generate the data of three users and encode to BPSK modulation and transmit through channel. (14)

Step: 2 Give the received data as input to the equalizer

TABLE I

CIRs FOR THE 3-USER 4-ANTENNA STATIONARY SYSTEM. SIMULATED CIRs WERE $c_{l,m}(z)/|c_{l,m}(z)|$ TO PROVIDE UNIT CHANNEL ENERGY

$c_{l,m}(z)$	$m=1$	$m=2$	$m=3$
$l=1$	$(-0.5+j0.4)+(0.7+j0.6)z^{-1}$	$(-0.1-j0.2)+(0.7+j0.6)z^{-1}$	$(-0.7+j0.9)+(0.6+j0.4)z^{-1}$
$l=2$	$(0.5-j0.4)+(-0.8-j0.3)z^{-1}$	$(-0.3+j0.5)+(-0.7-j0.9)z^{-1}$	$(-0.6+j0.8)+(-0.6-j0.7)z^{-1}$
$l=3$	$(0.4-j0.4)+(-0.7-j0.8)z^{-1}$	$(-0.1-j0.2)+(0.7+j0.6)z^{-1}$	$(0.3-j0.5)+(0.9+j0.1)z^{-1}$
$l=4$	$(0.5+j0.5)+(0.6-j0.9)z^{-1}$	$(-0.6-j0.4)+(0.9-j0.4)z^{-1}$	$(-0.6-j0.6)+(0.8+j0.0)z^{-1}$

Step: 3 Initialize the weight vectors

Step:4 Calculate the error and update the new weight

Step: 5 Give the received and target as input to the WIDROW-HOFF learning algorithm to calculate the next iteration weight. Go to step 4

Step: 6 Calculate the output obtained from the learning algorithm and plot the results.

Step: 7 STOP.

5. SIMULATION STUDY

The system used in our simulation supported M = 3 users with L = 4 receiver antennas [1-3], [12]. All three users had an equal transmit power. The ML = 12 CIRs are listed in Table I, each CIR having n_c = 2 taps. The CIRs used in the both the stationary and fading channels are extensions of the often-used single-input single-output (SISO) CIRs proposed by Proakis in his book, which were extended to the MIMO. The results obtained for the MUD through the channel with values shown in Table I are shown in Figures 3 to 5. In the simulation, all 12 CIRs were normalized to provide to unit channel energy, i.e., $\|c_{l,m}\|^2 = 1$ for all l and m. thus $SIR_i(m)$ = 0 dB for all m and i. each

Equalizer temporal filter had a length of n_F = 3, and the detector decision delay was chosen to be d = 1.

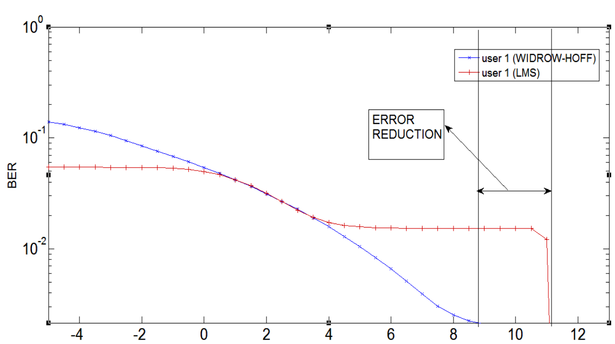

Fig.3 BER comparison of adaptive LMS and WIDROW-HOFF based MUD for 3 user and 4 antenna stationary system for user 1

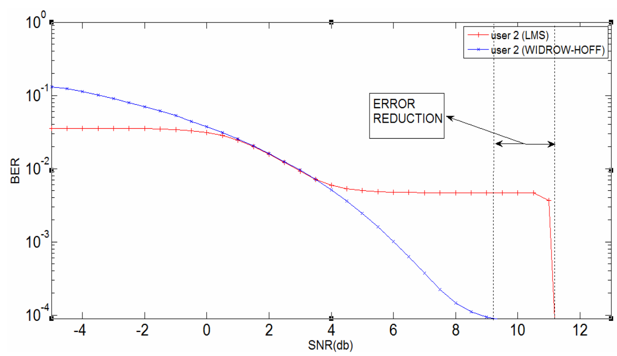

Fig. 4 BER comparison of adaptive LMS and WIDROW-HOFF based MUD for 3 user and 4 antenna stationary system of user 2

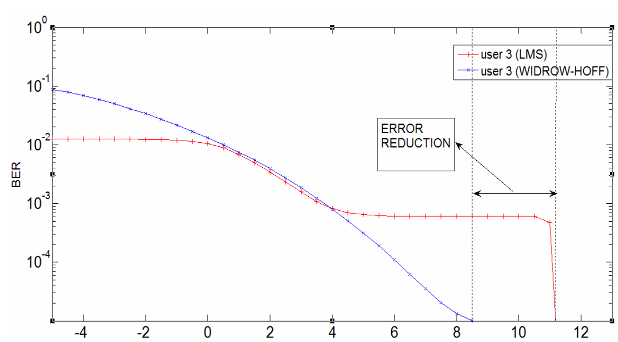

Fig. 5. BER comparison of adaptive LMS and WIDROW-HOFF based MUD for 3 user and 4 antenna stationary system of user 3

The step size of these algorithms is chosen for fast convergence and small steady state error as 0.001.

Fig. 3 shows the bit error rate of the both the WIDROW-HOFF and LMS methods for user one. Blue line represents the BER curve using WIDROW-HOFF and red line shows result LMS. In this figure LMS starts in between 10^{-0} to 10^{-1} and constant up to zero SNR after that graph is gradually decreasing up to four SNR and LMS follows constant up to eleven SNR and suddenly falls. But in the case of WIDROW-HOFF learning algorithm the graph is starts above from 10^{-1} and slope decreases to make the curve to reach to 8.5 SNR very quickly. Thus the error is reduced in this case with a margin of 2db.

Fig. 4 shows the bit error rate of both the WIDROW-HOFF and LMS methods for user two. In this case LMS starts in between 10^{-0} to 10^{-1} and constant up to zero SNR after that graph is gradually decreasing up to four SNR and LMS follows constant up to eleven SNR and suddenly falls. However, in the case of WIDROW-HOFF learning algorithm the graph is starts above from 10^{-1} and slope decreases to make the curve to reach to 9 SNR very quickly. In this case the reduction of error is 2db. The proposed algorithm showed its supremacy until this point. Finally, Fig. 5 shows the BER curve for user 3.

The output obtained by using WIDROW-HOFF and LMS algorithm shown in Fig. 5 is nearly similar to the other cases discussed previously. In this case LMS starts from 10^{-2} and constant up to zero SNR after that graph is gradually decreasing up to four SNR and LMS follows constant up to eleven SNR and suddenly falls. But in the case of WIDROW-HOFF learning algorithm the graph is starts below 10^{-1} and slope reaches to 8.1 SNR very quickly. The error reduction in this case is 2.9db. From all the results obtained it is seen that the proposed Widrow-Hoff algorithm for reducing BER is working efficiently. The results will look more pronounced if it plotted up to a SNR value of 15. However, due to vast time-taking process results only up to SNR of 12 has been shown. The comparison of net error reduction obtained by proposed algorithm and LMS algorithm is shown in Table II.

TABLE II
COMPARISON OF BER OF PROPOSED WIDROW-HOFF ALGORITHM AND LMS ALGORITHM

Users	WIDROW-HOFF (db)	LMS (db)	ERROR REDUCED (db)
User 1	8.5	11	2.5
User 2	9	11	2
User 3	8.1	11	2.9

6. CONCLUSIONS

MUD based on the neural network has been investigated for multiple-antenna-aided SDMA systems. A novel WIDROW-HOFF design has been proposed. It has been shown that WIDROW-HOFF assisted MUD can obtain significant performance gains over the standard LMS design, in terms of achievable system BER. It requires half of the computational complexity needed by the LMS algorithm for BPSK signaling. Our simulated results have demonstrated that the adaptive LMS assisted MUD converges faster and consistently achieves better BER performance, compared with the LMS STE-assisted MUD.

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