

## Distributional Convergence of Intermeeting Times Using Cluster Based Hybrid Random Walk Mobility Model

A.Sairam<sup>1</sup> and Mrs.M.Lavanya<sup>2</sup>

<sup>1</sup> Student, M.E – Computer Science & Engineering, Arulmigu Meenakshi Amman College of Engineering, Kanchipuram-604 410, TamilNadu, India. Email: a.sairam@yahoo.co.in.

<sup>2</sup> Supervisor, Asst Prof. Dept of Computer Science & Engineering, Arulmigu Meenakshi Amman College of Engineering, Kanchipuram-604 410, TamilNadu, India.

**Abstract**— The distribution of intermeeting times under the generalized Hybrid Random Walk mobility model. We show that when the conditional probability that two nodes can communicate directly with each other given that they are in the same cell is small and node's transitions in locations are independent over time, the distribution of intermeeting times can be well approximated using an exponential distribution. The mean of intermeeting times can be estimated using the number of cells in the network and the aforementioned conditional probability of having a communication link when the two nodes are in the same cell.

**Index Terms**— wireless communication, Stochastic processes.

### I. INTRODUCTION

Recently, there has been a growing interest in understanding the distribution of intermeeting times between mobile nodes in wireless networks (e.g., [1], [5], [10], [13]). An intermeeting time between two nodes refers to the amount of time during which they stay unable to communicate directly with each other after they lose the “communication link” between them. Since the ability of a (multihop) wireless network to transfer information between a pair of nodes in a timely manner depends critically on the (time-varying) network connectivity, understanding the statistical properties of intermeeting times is of much interest. Such an understanding is even more pressing in Disruption Tolerant Networks (DTNs) that rely on intermittent and/or sparse connectivity between nodes to forward information, in which we are primarily interested.

#### A. Short Survey of Relevant Literature

We summarize a few studies that are most relevant to this paper: Groenevelt et al. [9] studied the distribution of intermeeting times between two nodes under the popular Random Waypoint (RWP) mobility model and indicated that the distribution can be well approximated by an exponential distribution. Chaintreau et al. [4] examined several sets of traces collected in different settings and reported an

interesting observation that the empirical distributions exhibit a power law decay over a wide range (from a few minutes to a day or more). Karagiannis et al. [12], using additional sets of measurements, first illustrated the existence of a power law decay up to a certain point, which they call a characteristic time, followed by an exponential decay, hinting at a dichotomy in the empirical distributions of intermeeting times. Then, they demonstrated that such a dichotomy exists even under a simple Random Walk (RW) mobility model on a circle. An interesting study by Cai and Eun [3] suggests that, in most scenarios where the domain of mobility is bounded, the distribution is expected to have an exponential tail. A similar finding by Karagiannis et al. [12] also proves that when nodes move according to mutually independent irreducible Markov chains on a finite-state space, the distribution of intermeeting times is exponentially bounded. Cai and Eun also showed that when the domain is unbounded, a power law can emerge, indicating the possibility that a bounded domain used for simulation may be a main source of the emergence of an exponential tail in some cases. have been submitted for publication, should be cited as “unpublished” [4]. Papers that have been accepted for publication should be cited as “in press” [5]. In a paper title, capitalize the first word and all other words except for conjunctions, prepositions less than seven letters, and prepositional phrases. For papers published in translated journals, first give the English citation, then the original foreign-language citation [6].

#### B. Summary of Contributions

In this paper, we study the distribution of intermeeting times under a generalized Hybrid Random Walk (HRW) mobility model (described in Section 3). It is a generalization of the HRW mobility model first introduced by Sharma et al. [18], which includes the RW mobility model [6] and the independent and identically distributed (i.i.d.) mobility model used in [15] as special cases. We prove that, under this generalized HRW mobility model, as the conditional probability that two nodes can communicate directly with each other, given that they are in the same cell, decreases to zero, (suitably scaled) intermeeting times converge in distribution to an exponential rv.

This finding implies that when 1) the intensity of meetings between two nodes is sufficiently small and 2) node's transitions in location are independent over time, <sup>2</sup> the intermeeting times between them may be well approximated by exponential random variables (rvs). Our result allows heterogeneous mobility among the nodes and does not require that the network size grow unbounded. Moreover, the specifics of transition probabilities assumed in the mobility model affect only its parameter, but not the limiting distribution. These findings suggest that the distribution of intermeeting times is not sensitive to the details of nodes' mobility and may resemble an exponential distribution under a set of mild assumptions when the nodes' mobility is independent.

We also provide the intuition behind the finding; intermeeting times between two nodes in the generalized HRW mobility model can be represented as delayed geometric sums of independent rvs [11]. It is well known that a geometric sum of many i.i.d. rvs with a finite mean can be approximated using an exponential rv, which was first shown by Rényi [16]. Our finding follows from a generalization of Rényi's result to the case where the first summand in the geometric sum has a different distribution than the others.

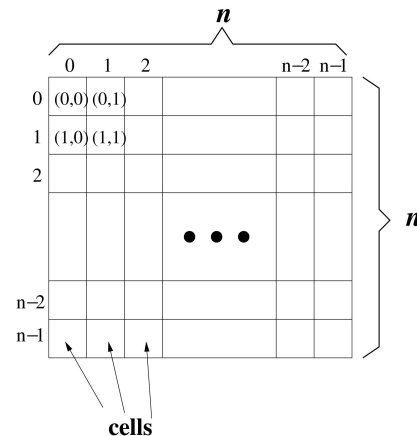
Note that, although we focus on the distribution of intermeeting times under the generalized HRW mobility model, the intuition behind our result is much more general and may be applicable to other mobility models; if the intermeeting times under some other mobility models can be approximated as a random sum of independent rvs where the rvs have similar, if not identical, distributions and the number of summands tends to be large and is roughly geometrically distributed, the distribution may still resemble an exponential distribution. In this sense, we use the generalized HRW mobility model as a concrete example of a larger class of mobility models with certain properties, under which our result will hold.

We emphasize that it is not our goal to disprove the power law decay or a dichotomy observed in empirical distributions of intermeeting times (e.g., [4], [12]). Instead, our goals are the following: First, as mentioned above, we illustrate that, when running simulation with a certain class of mobility models, including the generalized HRW mobility model, under which the intermeeting times may be approximated as delayed geometric sums of i.i.d. rvs, one can expect the distribution of intermeeting times to resemble an exponential distribution. Second, we provide additional insight into the emergence of limiting exponential distributions in some mobility models and how to estimate their parameters. We hope that these will enhance our growing understanding of the distribution

of intermeeting times under different sets of assumptions and settings, which is currently an active research area.

## II. BACKGROUND

In this section, we describe two previously proposed mobility models—the RW mobility model and the HRW mobility model. They are special cases of the generalized HRW mobility model we describe in the following section and under which we study the distribution of intermeeting times.



**Fig. 1. The RW mobility model.**

### A. Random walk mobility model

The RW mobility model was used by El Gamal et al. in [6] in the context of studying the scaling laws of the network transport throughput for multi-hop wireless networks. For each fixed  $n = 1, 2, \dots$ , a unit square area is divided into a discrete torus of size  $n \times n$ . Each of  $n^2$  rectangular areas is called a cell, and each cell is

identified by a pair  $(i, j)$ ,  $i, j \in \{0, 1, \dots, n-1\}$ , as

shown in Fig. 1.

Time is slotted into contiguous timeslots  $t = 0, 1, \dots$ . At timeslot  $t = 0$ , a node is initially placed in one of  $n^2$  cells according to some probability mass function (pmf). After its initial placement, a node in a cell, say  $(i, j)$ , first selects one of four adjacent cells, i.e., cells  $(i+1, j), (i-1, j), (i, j+1)$ , and  $(i, j-1)$ ,<sup>3</sup> with equal probability of  $1/4$  independently of the past,

and moves to the selected cell at timeslot  $t = 1$ . The node then repeats this process in every subsequent timeslot.

The location of a node at timeslot  $t = 0, 1, \dots$ , is denoted by  $C^{(n)}(t)$ , which indicates the cell where the node lies. From the description of the RW mobility model, it is clear that the discrete-time stochastic

process  $\{C^{(n)}(t); t = 0, 1, \dots\}$  is a time homogeneous Markov chain with state space  $\{(i, j) / i, j \in \{0, 1, \dots, n - 1\}\}$ .

### B. Hybrid Random walk mobility model

The HRW mobility model can be viewed as a generalization of the RW mobility model in the previous subsection [18]. It is parameterized by  $\beta$ ,  $0 \leq \beta \leq 1/2$ . For each fixed  $n = 1, 2, \dots$ , the unit square area is first divided into a discrete torus of  $n^\beta \times n^\beta$  cells. Each cell is then further divided into  $n^{(1-2\beta)/2} \times n^{(1-2\beta)/2}$  subcells. Thus, there are a total of  $n$  subcells. A subcell  $\ell(n)$  in the unit square area is uniquely identified by a pair  $\ell^{(n)} = (c^{(n)}, s^{(n)})$ , where  $c^{(n)} = (c_1^{(n)}, c_2^{(n)})$  with  $c_1^{(n)}, c_2^{(n)}$

$\in \{0, 1, \dots, n^\beta - 1\}$  specifies the cell to which the subcell  $\ell^{(n)}$  belongs, and  $s^{(n)} = (s_1^{(n)}, s_2^{(n)})$  with  $s_1^{(n)},$

$s_2^{(n)} \in \{0, 1, \dots, n^{(1-2\beta)/2} - 1\}$  designates the position of the subcell within the cell  $c^{(n)}$ .

The location of a node at timeslot  $t = 0, 1, \dots$ , is given by the subcell in which the node lies and is denoted by  $L^{(n)}(t) = (C^{(n)}(t), S^{(n)}(t))$ . Here,  $C^{(n)}(t) = (C_1^{(n)}(t), C_2^{(n)}(t))$  and  $S^{(n)}(t) = (S_1^{(n)}(t), S_2^{(n)}(t))$  are the cell and the subcell within  $C^{(n)}(t)$  where the node resides, respectively. The initial location  $L^{(n)}(0)$  of the node at timeslot  $t = 0$  is selected as follows: First, a cell  $C^{(n)}(0)$  is chosen according to some pmf. Then, one of the subcells in the cell  $C^{(n)}(0)$  is selected according to the discrete uniform distribution over the set of  $n^{1-2\beta}$  subcells in the cell.

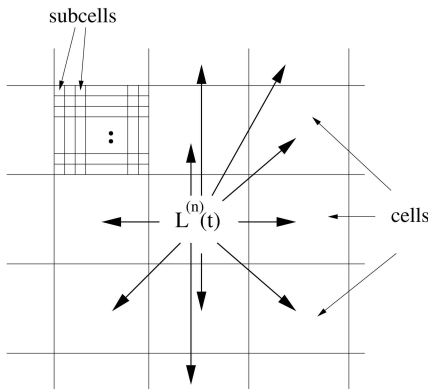


Fig. 2. The generalized HRW mobility model.

The transition of a node from one subcell at timeslot  $t = 0, 1, \dots$ , to another subcell at timeslot  $t+1$  is described by the following: A node located at subcell  $\ell^{(n)}$  at timeslot  $t$  first selects one of four adjacent cells with equal probability of  $1/4$  (as in the RW mobility model). Then, it chooses one of the subcells in the selected adjacent cell with equal probability of  $n^{-(1-2\beta)}$ , independently of the past and the selected cell. Hence,

$$\mathbf{L}^{(n)} := \{L^{(n)}(t); t = 0, 1, \dots\} \\ = \{(C^{(n)}(t), S^{(n)}(t)); t = 0, 1, \dots\},$$

Which we call the *trajectory* of the node, is a discrete-time stochastic process where  $C^{(n)} := \{C^{(n)}(t); t = 0, 1, \dots\}$  evolves according to the RW mobility model (hence is a time homogeneous Markov chain)

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independent because the subcells are selected independently of the past and selected cells as explained earlier.

When  $\beta = 0.5$ , the HRW mobility model reduces to the usual RW mobility model since there is only one subcell in each cell. On the other hand, when  $\beta = 0$ , a node moves according to the i.i.d. mobility model used in [15]. This is because there is only one cell consisting of  $n$  subcells and the node selects one of the subcells with equal probability  $n^{-1}$  in each timeslot, independently of the past.

## III. GENERALIZED HRW MOBILITY MODEL & INTERMEETING TIMES

### A. Hybrid Random walk mobility model

In the rest of this paper we consider a generalized HRW mobility model described in this subsection: For each fixed  $n = 1, 2, \dots$ , the unit square area is divided into a discrete torus of  $h_1(n) \times h_2(n)$  cells. Each cell is then further divided into  $h_2(n) \times h_2(n)$  subcells. Both  $h_1(n)$  and  $h_2(n)$  are assumed to be positive integers. It is clear that the total number of subcells is

$$(h_1(n) \times h_2(n))^2 =: N(n). \text{ Let } C^{(n)} = \{(i, j) \mid i, j \in \{0, 1, \dots,$$

$h_1(n) - 1\}\}$  be the set of cells and  $S(n) = \{(a, b) \mid a, b$

$\in \{0, 1, \dots, h_2(n) - 1\}\}$  be the set of subcells in a cell.

A node  $i$  moves on the discrete torus as follows: Let  $\{\Delta C_1^{(n)}(t); t = 0, 1, \dots\}$  be a sequence of i.i.d. rvs

with some pmf  $P_{c,i}^{(n)}$  over the set  $\{(i, j) \mid i, j \in$

$$\{-\lfloor (h_1(n) - 1)/2 \rfloor, \dots, \lfloor (h_1(n) - 1)/2 \rfloor\} =: \Delta C^{(n)}.$$

When node  $i$  is in cell  $C_i^{(n)}(t)$  at timeslot  $t = 0, 1, \dots$ , it first selects the cell  $C_i^{(n)}(t+1) = C_i^{(n)}(t) + \Delta C_i^{(n)}(t)^4$  and then picks one of the subcells,  $S_i^{(n)}(t+1)$ , in cell  $C_i^{(n)}(t+1)$  according to some pmf  $P_{s,i}^{(n)}$  over the set  $S^{(n)}$  of subcells in a cell<sup>5</sup>, independently of the past and the selected cell  $C_i^{(n)}(t+1)$ . Then, node  $i$  moves to the chosen subcell  $(C_i^{(n)}(t+1), S_i^{(n)}(t+1))$  at timeslot  $t + 1$ .<sup>6</sup> This process is repeated in each of subsequent timeslots by node  $i$ .

It is clear that  $C_i^{(n)} = \{C_i^{(n)}(t); t = 0, 1, \dots\}$  is a time homogeneous Markov chain with the state space  $C^{(n)}$ , where the transition probabilities are determined by the pmf  $P_{c,i}^{(n)}$ . The HRW mobility model is a special case of this generalized HRW mobility model with the

probability  $P_{c,i}^{(n)}(\Delta c)$ ,  $\Delta c = (\Delta c_1, \Delta c_2) \in \Delta C^{(n)}$ , equal to

$1/4$  if  $\|\Delta c\|_1 = |\Delta c_1| + |\Delta c_2| = 1$ , where  $\|\cdot\|$  denotes the  $L^1$ -norm, and  $0$  otherwise.

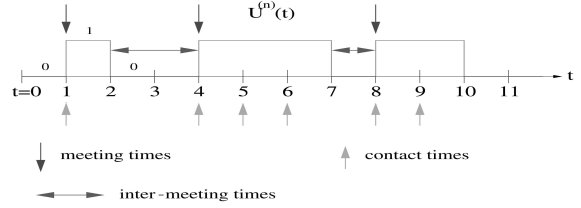
This mobility model allows node  $i$  to remain in the same *subcell* for more than one timeslot if  $P_{c,i}^{(n)}((0, 0)) = \Pr[\Delta C_i^{(n)}(t) = (0, 0)] > 0$ . Moreover, when  $P_{c,i}^{(n)}(\Delta c) >$

$0$  for all  $\Delta c \in \Delta C^{(n)}$ , node  $i$  located in some cell  $C_i^{(n)}(t)$  at

timeslot  $t$  can transition to any cell in  $C^{(n)}$  at timeslot  $t + 1$ . However, unlike in the i.i.d. mobility model [15], the probability with which a cell is selected for the following timeslot can depend on the current location of the node, retaining some memory.

#### B. Intermeeting Times Between Two Nodes

For each  $n = 1, 2, \dots$ , we have two nodes  $i = 0, 1$ , moving according to the generalized HRW mobility model on a discrete torus with  $N(n)$  subcells as described in the previous subsection. The pmfs  $P_{c,i}^{(n)}$  and  $P_{s,i}^{(n)}$ ,  $i = 0, 1$ , are not necessarily identical, allowing heterogeneous mobility among the nodes.



**Fig. 3. Plot of indicator functions  $U^{(n)}(t)$ ,  $t = 0, 1, 2, \dots$**

The location of node  $i$  at time  $t = 0, 1, \dots$ , is identified by the subcell  $L_i^{(n)}(t) = (C_i^{(n)}(t), S_i^{(n)}(t))$  at which the node is located. As explained in subsection

2.2,  $C_i^{(n)}(t) \in C^{(n)}$  and  $S_i^{(n)}(t) \in S^{(n)}$  denote the cell and

the subcell within  $C_i^{(n)}(t)$ , respectively, of node  $i$ 's location at timeslot  $t$ . The trajectory of node  $i = 0, 1$ , is given by

$$L_i^{(n)} := \{L_i^{(n)}(t); t = 0, 1, \dots\} \\ = \{(C_i^{(n)}(t), S_i^{(n)}(t)); t = 0, 1, \dots\}.$$

The stochastic processes  $L_i^{(n)}$ ,  $i = 0, 1$ , are assumed mutually independent.

#### IV. DISTRIBUTIONAL CONVERGENCES OF INTERMEETING TIMES UNDER THE GENERALIZED HRW MOBILITY MODEL

In this section we examine the distribution of the inter-meeting times  $I^{(n)}(k)$ ,  $k \geq 2$ , between two nodes under the generalized HRW mobility model. In particular, we are interested in their distribution when the two nodes meet *infrequently* as we focus on DTNs in which one-hop connectivity is often assumed sparse.

For each  $n = 1, 2, \dots$ , define  $\gamma^{(n)} := \sum_{s \in S^{(n)}} (P_{s,0}^{(n)}$

$\times P_{s,1}^{(n)}(s))$ ,<sup>8</sup> which is the probability that the two nodes are in contact, conditional on the event that they are in the same cell. In other words,

$$\gamma^{(n)} = \Pr [L_0^{(n)}(t) = L_1^{(n)}(t) \mid C_0^{(n)}(t) = C_1^{(n)}(t)] \\ = \Pr [S_0^{(n)}(t) = S_1^{(n)}(t)], t = 0, 1, \dots,$$

where the second equality follows from the assumed independence between  $C_i^{(n)}(t)$  and  $S_i^{(n)}(t)$ .

We introduce the following on the Markov

chains  $C_i^{(n)}$ ,  $i = 0, 1$ , are irreducible and aperiodic. (ii)  $\gamma^{(n)} > 0$  for all  $n = 1, 2, \dots$ , and  $\lim_{n \rightarrow \infty} \gamma^{(n)} = 0$ .

Since the state space  $C^{(n)}$  is finite for every  $n = 1, 2, \dots$ , Assumption 1(i) implies that the Markov chains  $C_i^{(n)}$ ,  $i = 0, 1$ , are also positive recurrent and, hence, ergodic. Therefore, it does not allow the case where the probability of staying in the same cell is one, i.e.,  $P_{c,i}^{(n)}((0,0)) < 1$ . Furthermore, starting from any initial locations  $L_i^{(n)}(0)$ ,  $i = 0, 1$ , the two nodes will arrive at the same cell at some finite  $t$  with probability one can verify that the unique stationary distributions  $\pi_i^{(n)}$  of the Markov chains  $C_i^{(n)}$  under the assumed ergodicity are the uniform distribution over the state space  $C^{(n)}$ . This in turn tells us that, starting from *any* cell, the expected number of timeslots it takes to come back to the same starting cell, is equal to the number of cells  $(h_1(n))^2$  in the network

When  $\gamma^{(n)} = 0$ , the two nodes never meet with probability one. Thus, in order to ensure that the two nodes will eventually meet with probability one, we need to assume  $\gamma^{(n)} > 0$ . Moreover, in order to study the distribution when two nodes meet infrequently, we study the asymptotic distribution (under appropriate scaling) as the frequency or intensity of meetings decreases with  $n$ , i.e.,  $\gamma^{(n)} \rightarrow 0$ . Note that Assumption 1(ii) can be satisfied with a *bounded* number of subcells in the network, i.e., there exists finite  $N$  such that  $N(n) \leq N$  for all  $n = 1, 2, \dots$ . In fact, we can have a *fixed* number of (cells and) subcells in the network

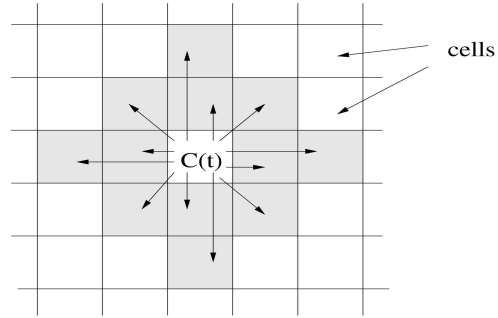
## V. INDEPENDENCE OF NODES' MOBILITY

Although we assumed that the trajectories of the two nodes  $L_i^{(n)}$ ,  $i = 0, 1$ , are mutually independent throughout, we can relax this assumption as follows: Suppose that when two nodes meet, they coordinate their movements so that they can stay in contact while exchanging information. During this period they may not follow the generalized HRW mobility model. Once they complete the transfer of message(s), they resume following the generalized HRW mobility model, independently of each other, until they meet again, at which point they repeat the process. It is clear that, under this assumption, the distribution of the inter-meeting times remains the same as before, whereas the number of consecutive timeslots they spend in contact after a meeting may change.

## VI. SIMULATION

In this section we simulate the generalized HRW mobility model with two nodes and study the empirical distribution of the inter-meeting times. We demonstrate that, although our finding in Theorem 1 is an asymptotic result obtained as  $\gamma^{(n)} \rightarrow 0$ , even for a non-negligible value of the conditional probability  $\gamma^{(n)}$  the distribution of inter-meeting times closely resembles an

exponential distribution with a good match between the predicted and empirical parameters.



**Fig. 4. Transition between cells by a node under the generalized HRW mobility model.**

Two nodes move according to the generalized HRW mobility model on a unit square area divided into  $49 = (h_1)^2$  cells.<sup>12</sup> Each cell is then further divided into  $9 = (h_2)^2$  subcells. We assume that the pmf  $P_{c,i}(\Delta c), \Delta c \in \{(i,j) | i, j \in \{0, \pm 1, \pm 2, \pm 3\}\}$  for selecting a next cell, is equal to  $1/12$  if  $1 \leq \|\Delta c\| \leq 2$  and 0 otherwise. In other words, a node in cell  $C(t)$  at timeslot  $t$  moves to one of the 12 shaded cells in Fig. 4 with equal probability of  $1/12$  at timeslot  $t+1$ . We use the discrete uniform distribution for subcell selection with  $P_{s,i}(s) = 1/9$ ,  $s \in \{(a,b) | a, b \in \{0, 1, 2\}\}$ , yielding the conditional probability  $\gamma = 1/9$ , which is not negligible.

## VI. CONCLUSION

The distribution of inter-meeting times under the generalized HRW mobility model. We showed that when the conditional probability that two nodes are in contact given that they are in the same cell is small, the inter-meeting times can be written as a delayed geometric sum of many independent rvs. This in turn implies that the distribution of inter-meeting times is well approximated by an exponential distribution even under heterogeneous mobility of the nodes. Moreover, the details of transition probabilities between cells and selection of sub cells change only the parameter of the limiting exponential distribution, but not the qualitative result (i.e., distributional convergence to an exponential distribution). These findings indicate that the distribution of inter-meeting times is insensitive to the details of nodes' mobility, and an exponential

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an asymptotic analysis, simulation results suggest that even for no negligible values of the aforementioned conditional probability, an exponential distribution offers a good approximation. Borrowing from the

existing literature, we also provided the intuition behind the emergence of a limiting exponential distribution.

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