International Journal of Review and Research in Applied Sciences and Engineering (IJRRASE)
Vol3 No.1. PP 18-22 March 2013 ISSN: 2231 – 0061X

Fuzzy Transportation Problem Using Triangular Membership Function-A New approach

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Abstract

In this Paper, a "Fuzzy Transportation Problem" is investigated using Triangular membership function. Fuzzy vogel's approximation method is used to obtain initial basic feasible solution and optimum solution is obtained using fuzzy modified distribution method. The method is illustrated by an example.

Key Words: **triangular fuzzy numbers, fuzzy triangular membership function, fuzzy Vogel's approximation method, fuzzy modified distribution method.**

I. INTRODUCTION

The transportation problem is a special class of linear programming problems. In a typical problem a product is to be transported from several sources to numerous locations at minimum cost. Suppose that there are 'm' warehouses where a commodity is stocked and transported to 'n' markets where it is needed. Let the availability in the warehouses be $a_1, a_2, ...a_m$ and the demands at the markets be $b_1, b_2, ...b_n$ respectively. In addition there is a penalty c_{ij} associated with transporting unit of product from the source i to the destination j. This penalty may be the cost or delivery time or safety of delivery etc. A variable x_{ij} represents the unknown quantity to be shipped from the source i to the destination j. The basic transportation problem was originally stated in [4], and later discussed in [7]. A linear programming problem using L-R fuzzy number was given in [9], an operator theory of parametric programming for Generalized Transportation Problem (GTP) was presented by [1]. In [2] the concept of decision making in fuzzy environment is proposed and in [3] the situation where all the parameters are fuzzy is considered. In [5], H.Isermann introduced an algorithm for solving this problem which provides an effective solution based on interval and fuzzy coefficients. Further development on Triangular membership functions in solving Transportation problem under fuzzy environment had been elaborated by [4, 10].

This paper is organized as follows; in section (2), some basic definitions on fuzzy set theory are listed. In section (3), the method of fuzzy modified distribution to find out the optimal solution for the total fuzzy transportation of minimum cost is discussed. In Section (4), a numerical example is worked out.

II. FUZZY BASCIS

2.*1. Definition* [1], If X is a collection of objects, and a fuzzy subset $\overline{A}$ of X is a set of ordered pairs. i.e. $\overline{A} = \{(x, \mu_A(x) / x \in X\}$, where $\mu_A(x)$ is called the membership function for the fuzzy set $\overline{A}$ .The membership function maps each element of X to a membership grade (or) membership value between 0 and 1

2.2. Definition [8], The α cut (or) α-level set of fuzzy subset $\overline{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. i.e., $\overline{A_\alpha} = \{x / \mu_A(x) \geq \alpha\}$

2.3. Definition [8], It A triangular fuzzy number is represented with the three points as $\overline{A} = (a_1, a_2 .a_3)$. This representation is interpreted as membership function and holds the following conditions,

(i) a_1 *to* a_2 is increasing function

(ii) a_2 *to* a_3 is decreasing function

(iii) $a_1 \leq a_2 \leq a_3$

$$\mu_{\overline{A}}(x) = \begin{cases} 0 & for \quad x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & for\, a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\, a_2 \leq x \leq a_3 \\ 0 & for \quad x > a_3 \end{cases}$$

2.3 *Definition* [8] let $a = [a_1, a_2, a_3]$ and $b = [b_1, b_2, b_3]$ be two triangular fuzzy numbers then the arithmetic operation is

Addition: $a + b = \{[a_1 + b_1], [a_2 + b_2], [a_3 + b_3]\}$ Subtraction: $a - b = \{[a_1 - b_3], [a_2 - b_2], [a_3 - b_1]\}$

Multiplication: $a * b = \{\min(a_1b_1, a_1b_2, a_3b_2, a_3b_3), a_2b_2, \max(a_3b_3, a_3b_1, a_1b_3, a_1b_1)\}$

2.4*Definition*

(i) Fuzzy feasible solution: If $x_{ij} > 0$, which satisfies the row and column sum is a fuzzy feasible solution.

(ii) Fuzzy basic feasible solution: A feasible solution is a fuzzy basic feasible solution if the number of non-negative allocation is atmost (m+n-1) where m is the number of rows and n is the number of columns in the transportation table.

(iii) Fuzzy non-degenerate basic feasible solution: A Fuzzy feasible solution to a transportation problem containing m origins and n destinations is said to be fuzzy non-degenerate, if it contains exactly (m+n-1) occupied cells with non zero values.

(iv) Fuzzy degenerate basic feasible solution: If a Fuzzy basic feasible solution contains less than (m+n-1) non-negative allocation, it is said to be degenerate.

III. FUZZY TRANSPORTATION PROBLEM:

Consider a Fuzzy Transportation problem with m fuzzy origins (row) and n fuzzy destinations (columns).Let $c_{ij} = [\underline{c_{ij}}, c_{ij}, \overline{c_{ij}}]$ is the cost of transportation one unit of the product from i[th] fuzzy origin to j[th] destinations,

$a = [a_1, a_2, a_3]$ be the quantity of commodity at fuzzy origin i,

$b = [b_1, b_1, b_1]$ be the quantity of commodity required at fuzzy destinations j, X_{ij} is quantity transported from i[th] fuzzy origin to j[th] fuzzy destinations. The linear programming model representing the Fuzzy Transportation problem is given by

$$\min z = \sum_1^m \sum_1^n [\underline{c_{ij}}, c_{ij}, \overline{c_{ij}}] [\underline{X_{ij}}, X_{ij}, \overline{X_{ij}}]$$

Subject to the constraints

$$\sum_{j=1}^{n} [\underline{X_{ij}}, X_{ij}, \overline{X_{ij}}] = [a_1, a_2, a_3]$$

, For i= 1, 2, 3...m rows,

$$\sum_{j=1}^{n} [\underline{X_{ij}}, X_{ij}, \overline{X_{ij}}] = [b_1, b_2, b_3]$$

For j = 1, 2, 3...n columns, for all $X_{ij} > 0$, the given fuzzy transportation problem is said to be balanced if $\sum_1^m a_i = \sum_1^n b_j$.

3.1Solution of Fuzzy Transportation Problem:

The Solution of FTP can be solved in two stages. (i) Initial Solution (ii) Optimal Solution. For finding the initial solution of FTP, fuzzy Vogel's approximation method is preferred over the other methods. Since the initial fuzzy basic feasible solution obtained, by this method is either optimal (or) very closed to the optimal solution. We are going to discuss fuzzy Vogel's approximation method and verify its solution in the nature of fuzzy triangular membership function.

3.2Fuzzy Vogel's Approximation Method Algorithm:

Step (i). Find the fuzzy penalty cost, namely the fuzzy difference between the smallest and next smallest fuzzy costs in each variable in each row and each column.

Step (ii). Among the fuzzy penalties found in step (i), choose the fuzzy maximum penalty by ranking method. If this maximum penalty is attained in more than one cell choose any one arbitrary.

Step (iii). If the selected row (or) column by step (ii), find out the cell having the least fuzzy cost by using the fuzzy ranking method. Allocate to this cell as much as possible depending on the fuzzy capacity and fuzzy demands.

Step (iv). Delete the row (or) column which is truly exhausted. Again compute column and row fuzzy penalties for the reduced fuzzy transportation table and then go to step (i) Repeat the procedure until all the demands are satisfied.

Once the initial fuzzy feasible solution is computed the next step in the problem is to determine whether the solution obtained is optimal or not. Fuzzy optimality test can be conducted to any fuzzy initial basic feasible solution of a fuzzy transportation problem provided such allocation has exactly (m + n-1) non-negative allocation, where "m" is the number of fuzzy origins and "n" is the number of fuzzy destinations. These allocation cells are independent. The optimality procedure is given below,

3.3 Fuzzy Modified Distribution Method:

This proposed method is used for finding the optimal basic feasible solution in the fuzzy environment and the following step by step procedure is used to find out the same.

Step 1. Find out the set of fuzzy triangular numbers $[\underline{u}_i, u_i, \overline{u}_i]$ and $[\underline{v}_i, v_i, \overline{v}_i]$ for each row and column satisfying $[\underline{u}_i, u_i, \overline{u}_i]$+ $[\underline{v}_i, v_i, \overline{v}_i]$=$[\underline{c}_{ij}, c_{ij}, \overline{c}_{ij}]$ for each occupied cell. To start with, we assign a fuzzy zero to any row (or) column having maximum number of allocations. If this maximum number of allocations is more than one, choose any one arbitrary.

Step 2. For each empty (unoccupied) cell, find $[\underline{u}_i, u_i, \overline{u}_i]$ and $[\underline{v}_i, v_i, \overline{v}_i]$

Step 3. Find out for each empty cell the net evaluation;

$$[\underline{z}_{ij}, z_{ij}, \overline{z}_{ij}] = [\underline{c}_{ij}, c_{ij}, \overline{c}_{ij}] - [\underline{u}_i, u_i, \overline{u}_i] + [\underline{v}_i, v_i, \overline{v}_i].$$

This step gives the optimality:

(a) if $Z_{ij} > 0$, the solution is optimal and a unique solution exists.

(b) if $Z_{ij} \geq 0$, the solution is optimal, but an alternate solution exists.

(C) if $Z_{ij}, < 0$, the solution is not fuzzy optimal. In this case, we go to next step, to improve the solution and minimize the cost.

Step 4. Select the empty cell having the most non negative value Z_{ij} from this cell we draw a closed path drawing horizontal and vertical lines with corners being occupied cells. Assign signs positive and negative alternatively and find out the fuzzy minimum allocation cell having negative sign. This allocation should be added to the allocation of the other cells having negative sign.

Step 5. The above step yields a better solution by making one (or) more occupied cell as empty cells. For the new set of fuzzy basic feasible allocation, repeat from step 1, till a fuzzy optimal basic feasible solution is obtained

4.Numerical Example

To solve the following fuzzy transportation problem of minimal cost, we start with the initial fuzzy basic feasible solution obtained by Fuzzy Vogel's Approximation method for which fuzzy cost and fuzzy requirement table is given below, the given problem is balanced fuzzy transportation problem, and the supply and demand costs are symmetric fuzzy triangular numbers (FTN).

***Table 4.1* The basic fuzzy transportation problem**

	D_1	D_2	D_3	D_4	Fuzzy capacity
O_1	[-2,0,2]	[0,1,2]	[-2,0,2]	[-1,0,1]	[0,1,2]
O_2	[4,8,12]	[4,7,10]	[2,4,6]	[1,3,5]	[2,4,6]
O_3	[2,4,6]	[4,6,8]	[4,6,8]	[4,7,10]	[4,6,8]
Fuzzy demand	[1,3,5]	[0,2,4]	[1,3,5]	[1,3,5]	

Since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ = [6, 11, 16] = [3, 11, 19] =33, the problem is balanced fuzzy transportation problem. There exists a fuzzy initial basic feasible solution.

***Table 4.2* It represents the initial basic fuzzy feasible solution, by VAM method**

	D_1	D_2	D_3	D_4	Fuzzy capacity
O_1	[-2,0,2]	[0,1,2] [0,1,2]	[-2,0,2]	[-1,0,1]	[0,1,2]
O_2	[4,8,12]	[4,7,10]	[2,4,6] [-3,1,5]	[1,3,5] [1,3,5]	[,2,4,6]
O_3	[2,4,6] [1,3,5]	[4,6,8] [-2,1,4]	[4,6,8] [-4,0,6]	[4,7,10]	[4,6,8]
Fuzzy demand	[1,3,5]	[0,2,4]	[1,3,5]	[1,3,5]	

Since, the number of occupied cell is m+n-1 = 6 and the occupied cells are independent. The initial solution is a non-degenerate fuzzy basic feasible solution. Therefore, the initial fuzzy transportation minimum cost is

$[\underline{z}_{ij}, z_{ij}, \overline{z}_{ij}]$ = [-27, 23,169] ------- (I)

4.3Moving to the optimal solution:

Here C_{ij} are fuzzy cost coefficient and X_{ij} (i = 1,2,3 , j = 1,2,3,4) are fuzzy allocations.

We have to find fuzzy membership functions of C_{ij} and X_{ij} for each cell (i, j).The membership function of fuzzy transportation cost $\mu_{C12}(x)$ is found as follows,

$$\mu_{C12}(x) = \begin{cases} \dfrac{x-0}{1-0} & 0 \leq x \leq 1 \\ \dfrac{x-2}{2-1} & 1 \leq x \leq 2 \end{cases}$$

To compute the interval of confidence for each level α, the triangular shapes will be described by functions of α in the following manner,

International Journal of Review and Research in Applied Sciences and Engineering (IJRRASE)
Vol3 No.1. PP 18-22 March 2013 ISSN: 2231 – 0061X

Here $\alpha = (x_1^{(\alpha)} - 0) \Rightarrow x_1^{(\alpha)} = \alpha$

$\alpha = (x_2^{(\alpha)} - 2) \Rightarrow x_2^{(\alpha)} = \alpha + 2$

$C_{12} = [x_1^{(\alpha)}, x_2^{(\alpha)}] = [\alpha, \alpha+2]$ -------- (i)

The membership functions of fuzzy allocation to the cell (i,j) $\mu_{X12}(x)$ as follows,

$$\mu_{X12}(x) = \begin{cases} \dfrac{x-0}{1-0} & 0 \le x \le 1 \\ \dfrac{x-2}{2-1} & 1 \le x \le 2 \end{cases}$$

Here $\alpha = (x_1^{(\alpha)} - 0) \Rightarrow x_1^{(\alpha)} = \alpha$

$\alpha = (x_2^{(\alpha)} - 2) \Rightarrow x_2^{(\alpha)} = \alpha + 2$

$X_{12} = [x_1^{(\alpha)}, x_2^{(\alpha)}] = [\alpha, \alpha+2]$

------------ (ii)

From (i) and (ii) we get

$C_{12} \bullet X_{12} = [\alpha^2, (\alpha+2)^2]$

------------ (A)

Exactly in the similar way,

$C_{23} \bullet X_{23} = [(2\alpha+2)(4\alpha-3), (6-2\alpha)(5-4\alpha)]$

------------ (B)

$C_{24} \bullet X_{24} = [(2\alpha+1)^2, (5-2\alpha)^2]$

------------ (C)

$C_{31} \bullet X_{31} = [(2\alpha+2)(2\alpha+1), (5-2\alpha)(6-2\alpha)]$

------------ (D)

$C_{32} \bullet X_{32} = [(2\alpha+4)(3\alpha-2), (8-2\alpha)(4-3\alpha)]$

------------ (E)

$C_{33} \bullet X_{33} = [(2\alpha+4)(4\alpha-4), (8-2\alpha)(6-6\alpha)]$

------------ (F)

A +B+C+D+E+F gives the Fuzzy minimum cost:

$Z = [31\alpha^2 + 28\alpha - 27, 35\alpha^2 - 166\alpha + 166]$

------------ (II)

Solving the equations:

$31\alpha^2 + 28\alpha - 27 - X_1 = 0$

------------ (G)

$35\alpha^2 - 166\alpha + 166 - X_2 = 0$

------------ (H)

We get

$$\alpha = \frac{[-28 + \{(31)^2 - 124(-27 - X_1)\}^{1/2}]}{56},$$

$$\alpha = \frac{[166 + \{(166)^2 - 140(166 - X_2)\}^{1/2}]}{70}$$

$$\mu_{\min.\cos t.Z(x))} = \begin{cases} \dfrac{-28 + \{(28)^2 - 124(-27 - X)\}^{1/2}}{56} & -27 \le x \le 23 \\ \dfrac{166 + \{(166)^2 - 140(166 - X_2)\}^{1/2}}{70} & 23 \le x \le 169 \end{cases}$$

.

This is the required fuzzy membership function of fuzzy transportation minimum cost Z (using the equation number 1).

4.4 Determination of fuzzy optimal solution: Applying the fuzzy modified distribution method, we determine a set of triangular fuzzy numbers $[\underline{u}_i, u_i, \overline{u}_i]$ and $[\underline{v}_i, v_i, \overline{v}_i]$ each row and column such that $[\underline{c}_{ij}, c_{ij}, \overline{c}_{ij}]$ = $[\underline{u}_i, u_i, \overline{u}_i]$ + $[\underline{v}_i, v_i, \overline{v}_i]$ for each occupied cell. Since the 3rd row has maximum number of allocation, we give fuzzy number $[\underline{u}_3, u_3, \overline{u}_3]$ = [-1, 0, 1]. The remaining numbers can be obtained as given by:

$[\underline{v}_1, v_1, \overline{v}_1]$ = [3, 4, 5], $[\underline{v}_2, v_2, \overline{v}_2]$ = [3, 6, 7], $[\underline{v}_3, v_3, \overline{v}_3]$ = [3, 6, 7],

$[\underline{u}_2, u_2, \overline{u}_2]$ = [-5,-2, 3]. $[\underline{v}_4, v_4, \overline{v}_4]$ = [-2, 1, 0], $[\underline{u}_1, u_1, \overline{u}_1]$ = [-7, -5,-1]

We find for each empty cell the sum of $[\underline{u}_i, u_i, \overline{u}_i]$ and $[\underline{v}_i, v_i, \overline{v}_i]$.

Next, the net evaluation of $[\underline{z}_{ij}, z_{ij}, \overline{z}_{ij}]$ are found .Using the table below:

Table 4.5 It represents the unoccupied cells

	D_1	D_2	D_3	D_4	Fuzzy capacity
O_1	[-2,0,2] *[-6,-1,8]	[0,1,2] [0,1,2]	[-2,0,2] *[-8,-1,-6]	[-1,0,1] *[0,4,10]	[0,1, 2]
O_2	[4,8,12] *[-4,6,6]	[4,7,10] *[-6,3,12]	[2,4,6] [-3,1,5]	[1,3,5] [1,3,5]	[,2,4 ,6]
O_3	[2,4,6] [1,3,5]	[4,6,8] [-2,1,4]	[4,6,8] [-4,0,6]	[4,7,10] *[3,6,13]	[4,6, 8]
Fuzzy demand	[1,3,5]	[0,2,4]	[1,3,5]	[1,3,5]	

Since $[\underline{z}_{ij}, z_{ij}, \overline{z}_{ij}]$>0, the solution is fuzzy optimal and unique. Minimum cost of FTP is $[\underline{z}_{ij}, z_{ij}, \overline{z}_{ij}] = [-27, 23, 169]$

Using exactly in the same procedure as in the fuzzy initial basic feasible solution. We could find fuzzy membership function of optimal solution.

V. CONCLUSION

In this paper, we obtained an optimal solution for Fuzzy Transportation Problem of minimal cost using Fuzzy Triangular Membership Function. The new arithmetic operations of Triangular fuzzy numbers are employed to get the fuzzy optimal solutions. The optimal solution obtained by using the fuzzy modified distribution method can also be verified in the Triangular Membership Function. This would be a new attempt in solving the Transportation Problem in fuzzy environment. In our future extension we would like to utilize the new optimization techniques in the literature. Anticipating the valuable comments and suggestions,

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