

Viscous Dissipation Effect on Steady free Convection Flow past a Semi-Infinite Flat Plate in the presence of Magnetic Field

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Abstract:

The viscous dissipation effect in the presence of a magnetic field is thoroughly studied in the two-dimensional free convective flow past a continuously moving semi-infinite flat plate. The velocity and temperature profiles are plotted for various parameters such as Gebhart number (Gb), Prandtl Number (Pr) and the magnetic field (M). The equations are solved using Runge-Kutta method with shooting technique. It is observed that the normal behavior of Viscous Dissipation effect changes in the presence of a magnetic field.

Keywords: Heat transfer, Moving surface, Prandtl number, Gebhart number, (Gb) Viscous dissipation.

1. INTRODUCTION

A moving boundary involving fluid flow and its effects of viscous dissipation has found many industrial applications such as agriculture, petroleum industries, stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Sakisdas (1961) investigated the flow of a fluid in the boundary layer and revealed that the growth of the boundary layer is in the direction of motion of the solid surface. But it is different from Blasius flow past a plate. In 1962 Gebhart proposed that the act of gravitational force in the natural convection of fluid flow generates appreciable temperature, reveals the rate of change of mechanical energy which is converted into the heat per unit volume in a viscous fluid. Many research scholars have some constraints on velocity and temperature distribution on the surface. In 1967 Tsouetal investigated some theoretical and practical studies of flow and temperature fields in the boundary layer on a moving surface for various values of the Prandtl number.

In 1969, Gebhart and Mollendorf study the viscous dissipation effects on the free convection of fluids with the effect of exponential variation wall temperature. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite

isothermal vertical plate was studied by Soundalgekar et.al (1979). In 1995, M. Massoudi investigated the effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe. In 1997 P.V.S.N. Murthy and P. Singhare initiated the effect of viscous dissipation on non-darcy natural convection flow along an isothermal vertical wall in a porous medium. In 1999 Anjali and Kandasamy analysed the viscous dissipation effects on heat transfer over a fluid flow over a continuous moving semi-infinite flat plate. Electrically conducting fluid over a sphere in the presence of Magnetic field with the effect of viscous dissipation has been studied by Alam et.al (2007). Cortell (2007) investigated on viscous flow with effect of heat transfer over a non-linear stretching sheet. Moreover, the problem extended to study the effect of viscous dissipation and radiation on the thermal boundary layer over a non-linear stretching sheet by Cortell (2008). Before that in (2006) Raptis and Perdikis studied viscous flow near a non-linear stretching sheet in the presence of chemical reaction and magnetic field. In 2000, Nield studied the Resolution of Paradox involving Involving Viscous dissipation and nonlinear drag in a porous medium. The study of MHD natural convective flow of an incompressible viscous fluid over a infinite vertical oscillating plate by Kishan et.al (2006). In 2009 Arabauy studied the effects of mass transfer over a stretching surface in the presence of suction/injection effects. Before this in (2007) Alao and Adegbic studied the combined effect of viscous dissipation and radiation of non-Newtonian fluid over a non-darcy porous medium with natural convection flow. El-Anabawy (2009) investigated the effects of chemical reaction on mass transfer over a stretching surface. Kanri et.al (2011) study the combination effect of viscous dissipation and radiation on free convection in non-darcy porous medium saturated with non-Newtonian fluid. In 2011 Geetha Palansamy and M.B.K. Moorthy investigated that the Viscous dissipation effect on steady free convection flow and mass transfer flow past a semi-infinite flat plate.

The aim of the present study is to study the effect of viscous dissipation on heat transfer in the flow past a continuously moving semi infinite flat plate in the presence of magnetic field. The analysis showed that the viscous dissipation have significant influence on the non-dimensional heat transfer coefficients

II. MATHEMATICAL FORMULATION

Here, the two dimensional fluid flow past a continuously moving semi-infinite flat plate in the presence of magnetic field has been considered. Assume that x - axis to be taken along the flat plate and y – axis to be considered normal to the plate. Let u and v the velocities components of the fluid flow along x and y directions respectively. The governing equations under the boundary layer and Boussinesq approximations may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho x} U_0^2 \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 \quad (3)$$

Along with the boundary conditions:

$$u = U_0, v = 0, T = T_w, \text{ at } y = 0$$

$$u = 0, T \rightarrow T_\infty, \text{ at } y \rightarrow \infty$$

Here:

c_p = Specific heat

ν = Kinematic viscosity of the ambient fluid

U_0 = Velocity of fluid at $y=0$

B_0 = Magnetic field intensity

σ_0 = Thermal diffusivity

The following similarity transformation is used to converted the above partial differential equations into ordinary differential equation as follows:

$$\eta = y \sqrt{\frac{U_0}{\nu x}} \quad (4)$$

$$\psi(x, y) = \sqrt{\nu x U_0} f \quad (5)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \quad (6)$$

Where:

η = The similarity variable, θ = The dimensionless stream function depends on η only

Let Ψ be the stream function defined such that $u = \frac{\partial \Psi}{\partial y}$ $v = -\frac{\partial \Psi}{\partial x}$ so that, the equation of continuity automatically satisfied.

$$\frac{\partial u}{\partial x} = -\frac{U_0}{2} f'' \left(y \sqrt{\frac{U_0}{\nu}} x^{-3/2} \right) \quad (7)$$

$$\frac{\partial v}{\partial y} = y \frac{U_0}{2x} f'' \left(\sqrt{\frac{U_0}{\nu x}} \right) \quad (8)$$

The equation (7) and (8) automatically satisfied the equation (1)

From the equation of motion

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho x} U_0^2$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = f' U_0 \left(-\frac{U_0 y}{2x} f'' \sqrt{\frac{U_0}{\nu x}} \right) + \left(\frac{U_0}{2x} f' y - \sqrt{\frac{U_0 \nu}{2\sqrt{x}}} f \right) \left(U_0 f'' \sqrt{\frac{U_0}{\nu x}} \right)$$

$$= \left(\frac{U_0}{2x} f' y - \frac{\sqrt{\nu U_0}}{2\sqrt{x}} f \right) \left(U_0 f'' \sqrt{\frac{U_0}{\nu x}} \right)$$

$$(9) \quad \frac{\partial^2 u}{\partial y^2} = U_0^2 \frac{f'''}{\nu x} \quad (10)$$

Using equation (9) and (10) in equation (2), we get as

$$\left(\frac{U_0}{2x} f' y - \frac{\sqrt{\nu U_0}}{2\sqrt{x}} f \right) \left(U_0 f'' \sqrt{\frac{U_0}{\nu x}} \right) = \nu U_0^2 \frac{f'''}{\nu x} - \frac{\sigma_0 B_0^2}{\rho x} U_0^2$$

$$f''' + \frac{1}{2} f f'' - M = 0 \quad (11)$$

The Equation (11) is the similarity transformation of equation (2)

From the equation of energy

$$u \frac{\partial T}{\partial x} = -f' \theta' U_0 y \sqrt{\frac{U_0}{\nu}} \times \frac{1}{2} x^{-\frac{3}{2}} (T_w - T_\infty)$$

$$v \frac{\partial T}{\partial y} = \frac{U_0}{2x} f' y (T_w - T_\infty) \theta' \sqrt{\frac{U_0}{\nu x}} - \frac{\sqrt{\nu U_0}}{2\sqrt{x}} (T_w - T_\infty) \theta' \sqrt{\frac{U_0}{\nu x}} f$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = -f' \theta' U_0 y \sqrt{\frac{U_0}{\nu}} \frac{1}{2} x^{-\frac{3}{2}} (T_w - T_\infty) + \frac{U_0}{2x} f' y (T_w - T_\infty) \theta' \sqrt{\frac{U_0}{\nu x}} - \frac{\sqrt{\nu U_0}}{2\sqrt{x}} (T_w - T_\infty) \theta' \sqrt{\frac{U_0}{\nu x}} f$$

$$= -\frac{U_0}{2x} (T_w - T_\infty) \theta' f$$

$$\frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 = \frac{k}{\rho c_p} (T_w - T_\infty) \theta'' \frac{U_0}{vx} + \frac{U_0}{x} (T_w - T_\infty) G_b (f'')^2$$

$$-\frac{U_0}{2x} (T_w - T_\infty) \theta' f' = \frac{k}{\rho c_p} (T_w - T_\infty) \theta'' \frac{U_0}{vx} + \frac{U_0}{x} (T_w - T_\infty) G_b (f'')^2$$

$$\frac{k}{\rho c_p} (T_w - T_\infty) \theta'' \frac{U_0}{vx} + \frac{U_0}{x} (T_w - T_\infty) G_b (f'')^2 + \frac{U_0}{2x} (T_w - T_\infty) \theta' f' = 0$$

$$\frac{U_0 (T_w - T_\infty)}{x} \left\{ \frac{\theta' f'}{2} + \frac{k}{\rho c_p} \theta'' \frac{1}{v} + G_b (f'')^2 \right\} = 0$$

The final similarity transformation of equation of Energy is

$$\theta'' + \frac{1}{2} Pr f \theta' + G_b (f'')^2 Pr = 0 \quad (12)$$

The final transformed equations with boundary conditions are :

$$f''' + \frac{1}{2} f f'' - m = 0$$

$$\theta'' + \frac{1}{2} Pr f \theta' + G_b Pr (f'')^2 = 0$$

With the boundary and initial conditions as:

$$f(0) = 0; f'(0) = 1; \theta(0) = 1 \text{ at } \eta = 0$$

$$f'(\infty) = 0; \theta(\infty) = 0 \text{ at } \eta \rightarrow \infty \quad (13)$$

The non-dimensional numbers are defined as the Viscous dissipation parameter known as the Gebhart number, given by $G_b = \frac{U_0^2}{c_p(T_w - T_\infty)}$, and the Prandtl number given by $Pr = \nu/a$.

III. MATERIALS AND METHODS

The set of Eq. (11) & (12) together with the boundary conditions (13) have been solved numerically by applying shooting technique along with Runge-Kutta Gill method. The ordinary differential Eq.(1) to (3) along the boundary conditions are solved by giving approximate initial guess values for the missing initial conditions of $f(0)$, $\theta'(0)$, and these values are matched with the corresponding boundary conditions at $f'(\infty)\theta(\infty)$. Extensive calculations have been performed to obtain the flow and temperature fields for a wide range of parameters $0 < Pr \leq 10$, and $1 \leq G_b \leq 10$.

IV. DISCUSS OF RESULTS

For physical understanding of the problem numerical computations are carried out of different physical parameters Magnetic field, Gambhart Number, Prandtl Number upon the nature of the flow. The value of Prandtl number Pr is chosen such that they represent water ($Pr=7.0$). The numerical values of the velocity are computed for different physical parameters like Gambhart Number. The Velocity profile for different values of Gambhart Number are shown in figure .1

Fig.2 demonstrates viscous dissipation effects on temperature profile with different values of Gambhart Number with Magnetic field and Prandtl number. It observed viscous dissipation effects goes on decrease with increase value of Gambhart Number. Something were analyzed by giving the different values of magnetic field and prandtl number in the figure 3 & 4.

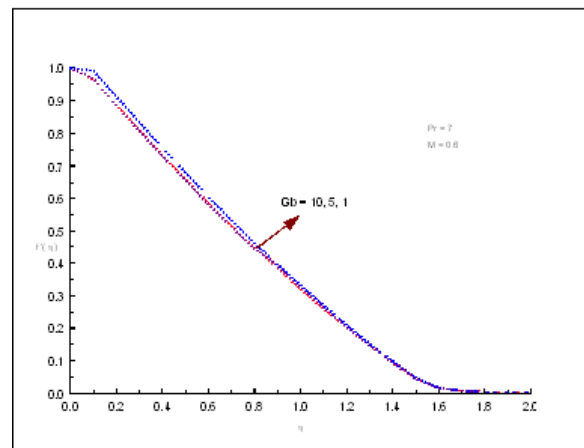


Figure.1 Effect of viscous dissipation parameter on no dimensional velocity f'

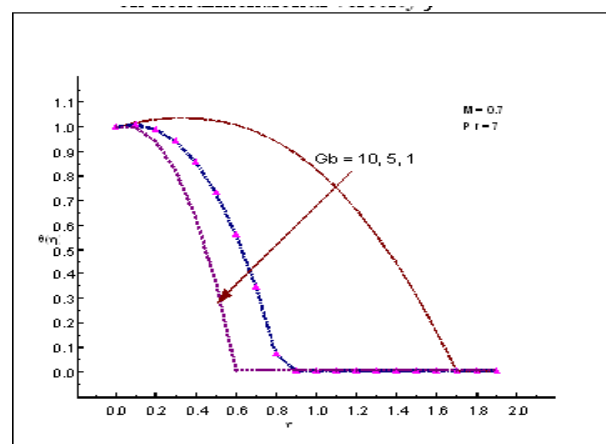


Figure.2 Effect of viscous dissipation parameter Gb on non-dimensional temperature θ

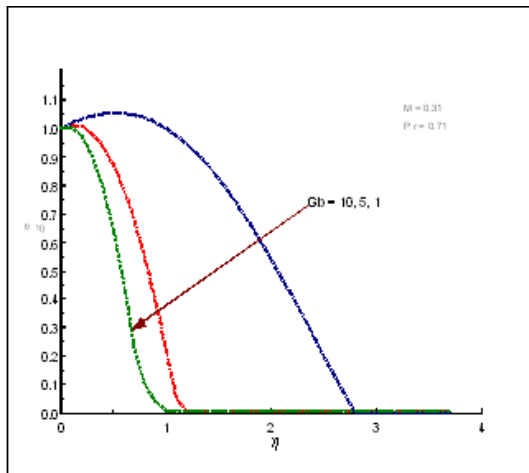


Figure.3.Effect of viscous dissipation parameter G_b on non-dimensional temperature θ

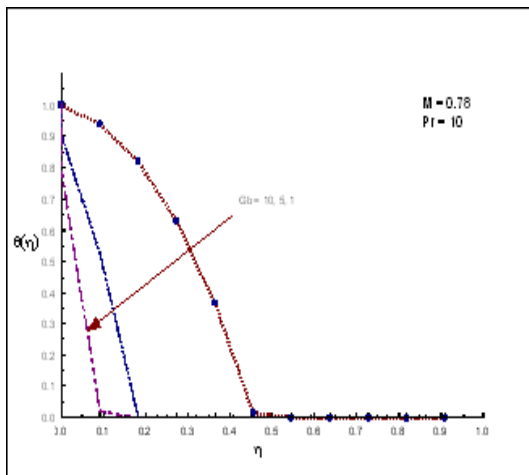


Figure-4 Effect of viscous dissipation parameter G_b on non-dimensional temperature θ

V.CONCLUSION

An exact analysis of Viscous Dissipation effect on steady free convection flow past a continuously moving semi-infinite flat plate in the presence of magnetic field has been studied. The similarity equations are solved by using runge-kutta gill method with shooting techniques. The effect of different parameters like magnetic field parameter, Pradtl number, Gebhart number are studied graphically. The conclusions of the study are as follows:

- Viscous dissipation effect increase in the velocity profile with increase value of Gebhart number(G_b)
- Viscous Dissipation effect of fluid flow in the temperature profile is decrease with increasing value of magnetic field.

➤ Viscous Dissipation effect of fluid flow in the temperature profile is decrease with the increase value of Pradtl number.

➤ Viscous Dissipation effect more in the air than in water.

REFERENCES

[1].Adegnie, K.S. and F.I. Alao, 2007. Flow of temperature-dependent viscous fluid between parallel heated walls : Exact analytical solutions in the presence of viscous dissipation. *J.Math, Stat.*, 3: 12-14. DOI: 10.3844 / JMSSP.2007.12.14.

[2].Alam, M.M., M.A., Alim and M.M.K. Chwdhury, 2007. Viscousdissipationeffects on MHD natural convection flow over a sphere in the presence of heat generation. *Nonlinear Analysis : Model. Control*, 12 : 447-459, http://www.lana.lt/journal/27/MdM_Alam.pdf

[3].Anjali, S.P. and R.Kandasamy, 1999, Effects of chemical reaction,heat and mass transfer on laminar flow along a semi infinite horizontal plate. *Heat.MassTrans.* 35 : 465-467, DOI : 10.1007/S002310050349

[4].Cortell, R, 2006. Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet. *Phys. Lett. A*, 357 : 298-305, DOI : 10.1016/J.PHYSLETA.2006.04.051.

[5].Cortell, R., 2007a. MHD flow and heat transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet subject with chemically reactive species. *Chemical Eng. Process.*, 46 : 721-728 : DOI : 10.1016/J.CEP.2006.09.008

[6].Cortell, R, 2007b. Viscous flow and heat transfer over a non-linearly stretching sheet, *Applied Math. Computer.*, 184 : 864-873. DOI : 10.1016/ J.AMC.2006.06.077

[7].Cortell, R., 2008. Effects of viscous dissipation and radiation on the thermal boundary layer ove a non-linearly stretching sheet. *Phys. Lett. A*, 372 :631-636. DOI : 10.1016/ J. PHYSLETA . 2007.08.005

[8].El-Arabawy.H.A.M., 2009, Exact solution of mass transfer over a stretching surface with chemical reactiojn and suction/injection.*J.Math. Stat.*,5:159-166.



DOI :10.3844/ JMSSSP. 2009. 159.166.

[9].Gebhart B., 1962. Effect of viscous dissipation in natural convection.J.Fluid.Mech., 14 : 225-232.

DOI : 10.1017 / S0022112062001196.

[10].Gebhart, B and Mollendorf, 1969. Viscous dissipation in external natural convection flows. J.Fluid. Mech., 38 : 97-107.

DOI : 10.1017/ S0022112069000061

[11].Kairi, R.R.,P.V.S.N. Murthy and C.O. Ng, 2011. Effect of viscous dissipation on natural convection in a non-arcy porous medium saturated with non-newtonian fluid of variable viscosity. Open Transport Phenomena J., 3. 1-8.

DOI : 10.2174/1877729501103010001

[12].Kishan, N. and P. Amrutha, 2010. Effects of viscous dissipation on MHD flow with heat and mass transfer ovre a stretching surface with heat source,thermal stratification and chemical reaction. J.Naval Archit. Marine Eng., 7 11-18.

DOI : 10.3329/jname.v7i;.3254

<http://www.banglajol.index.php/JNAME/article/view/3254>

[13].Kishan. N., Srihari and Rao, J.A., 2006. MHD free convective flow of an incompressible viscous dissipative fluid in an infinite vertical oscillating plate with constant heat flux. J.Energy Heat Mass Trans., 28 : 19-28.

<http://direct.bl.uk/bld/Place>

Order

do?UIN=200051073&ETOC=N&From
=Searchengine

[14].Kumar, H., 2009. Radiative heat transfer with hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. Thermal Sci., 13 : 163-169.

DOI : 10.2298/TSC10902163K

[15].Raptis, A. and C. Perdakis, 2006.Viscous flow over a non-linearly stretching sheet in the presence of a chemical reaction and magnetic field.Int. J.Non-Linear Mech., 41 : 527-529.

DOI : 10.1016/J.IJNONLINMEC. 2005.12.2003

[16].Sakiadis, B.C., 1961a. Boundary layer behaviour on continuous moving solid surfaces i. boundary layer equations fo two-dimensional and axi-symmetric flow. AIChE J., 7 : 26-28

DOI : 10,1002/aic.690070108

[17].Sakiadis, BC., 1961b. Boundary layer behaviour on continuous solid surfaces : II the boundary layer on a continuous flat surfaces. AIChE J., 7:221-225,

DOI : 10.1002/aic.690070211

[18].Subhas Abel. M., K.A. Kumar and R.Ravikumara 2011. MHD flow and heat transfer with effect of buoyancy, viscous and joules dissipation over a nonlinear vertical stretching porous sheet with partial slip. Sci. Res., 3 : 285-291.

DOI : 0.4236/Eng.2011.33033

[19].Tsou, F.K., E.M. sparrow and R.J. Goldstein, 1967. Flow and heat transfer in the boundary layers on a continuous moving surface Int. J.Heat Mass Trans., 10 : 219-235 :

DOI : 10.1016/0017-9310(67)90100-7

[20].Vajravelu, K and A. Hadjinalaou, 1993. Heat transfer in a visous fluid over a stretching sheet with viscous dissipation and internal heat generation. Int. Comm. Heat. Mass Trans., 20:417-430.

DOI : 10.1016/0735-1933(93)90026-R