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MEYNIEL'S GRAPHS ARE STRONGLY PERFECT

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ABSTRACT

The graph-theoretic notions used here are those of [3]. A graph is called *perfect* if the chromatic number of each of its induced subgraph H equals the number of vertices in the largest clique of H; it is called *strong perfect* [4] if each of its induced subgraphs H contains a stable set which meets all the maximal cliques in H. (Here, as usual, 'maximal' is meant with respect to set-inclusion.) It is easy to show that every strongly perfect graph is perfect. Meyniel[5] proved that a graph is perfect whenever each of its odd cycles of length at least five has at least two chords. The purpose of this paper is to strengthen Meyniel's result as follows:

Theorem:

If every odd cycle of length at least five in a graph G has at least two chords then G is strongly perfect.

The following useful observation has been made by Meyniel.

Lemma:

If a graph G = (V, E) contains an odd cycle $[x_0, x_1, x_2, \ldots, x_{2t}, x_0]$ such that the path $[x_1, x_2, \ldots, x_{2t}]$ is chordless and x_0 is non-adjacent to at least one x_k, then G contains an odd cycle of length at least five with at most one chord.

Proof:

If $x_0 x_2 \in E$ then consider the largest i such that x_0 is adjacent to $x_1, x_2, \ldots, x_i$ And the smallest j such that $j > i$ and $x_0x_j \in E$: the cycle $[x_0, x_i, \ldots, x_j, x_0]$ has no chords, the cycle $[x_0, x_{i-1}, x_i \ldots, x_j]$ has precisely one chord, and one of these two cycles is odd. If $x_0 x_2 \notin E$ then consider the smallest even j such that $x_0x_j \in E$ and the largest i such that $i \leq j-2$ and $x_0x_i \in E$: the odd cycle $[x_0, x_i, \ldots, x_j, x_0]$ has at most one chord.

By a starter in a graph G, we shall mean a cycle $wv_0v_1\ldots v_kw$ such that

(i) v_0 is adjacent to none of the vertices $v_2, v_3, \ldots, v_k$.
(ii) w is not adjacent to v_1.
(iii) Some stable set S, containing v_1 and v_k, meets all the maximal cliques in $G\text{-}v_0$

Lemma:

If a graph contains a starter then it contains an odd cycle of length at least five with at most one chord.

Proof:

We shall present an informal description of an efficient algorithm which, given any starter in G = (V, E), finds the desired cycle. No generality is lost by assuming that

(iv) $[v_1, v_2, \ldots, v_k, w]$ is the shortest path from v_1 to w with the next-to-last vertex in S and all the vertices except the first and the last non-adjacent to v_0.
In particular, (iv) implices that
(v) The path $[v_1, v_2, \ldots, v_k]$ has no chords.
Next, we may assume that
(vi) Every v_r adjacent to w has an even subscript r;
Otherwise the desired cycle can be found at once by applying lemma to the odd cycle $wv_{0,}\ldots v_rw$.
Write $y \in S^*$ if $y \in S$ and y is adjacent to two consecutive vertices v_j, v_{j+1} on the path $[v_0, v_1, v_2, \ldots, v_k]$. We may assume that
(vii) No $y \in S^*$ is adjacent to v_0;
Otherwise the desired cycle can be found at once by applying lemma to any odd cycle $[yv_0\ldots v_iy]$. Now it follows that
(viii) No $y \in S^*$ is adjacent to w;
Otherwise (iv) would be contradicted by $v_0v_1\ldots v_iyw$ such that i is the smallest subscript with $yv_i \in E$. Next,we may assume that
(ix) Each $y \in S^*$ is adjacent to at least three vertices on the path $v_0v_1\ldots v_k$;
Otherwise the desired cycle is $[w, v_r, \ldots v_j, y, v_{j+1}, \ldots, v_s, w]$ with r standing for the largest subscript such that $r \leq j$, $wv_r \in E$ and s standing for the smallest subscript such that $s \geq j+1$, $wv_s \in E$. Now it follows that
(x) Each $y \in S^*$ is adjacent to precisely three vertices v_{i-1} , v_i, v_{i+1} on the path $[v_0, v_1, \ldots, v_{k]};$
Otherwise (iv) would be contradicted by $v_1, \ldots v_ryv_s, \ldots v_k$ such that r is the smallest subscript with $yv_r \in E$ and s is the largest subscript with $yu_s \in E$.

Now observe that,for any $y \in S^*$ adjacent to v_{i-1}, v_1 and v_{i+1}, the substitution of y for v_i in the original starter yields a new starter with a smaller S^*.Repeating this operation as many times as possible, we eventually obtain a starter $w\bar{v}_0\bar{v}_1\ldots\bar{v}_kw$ satisfying (iv) and hhaving an empty S^*.

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